

that is distant from the actual ray by distances of the third order. If we calculate the integral for this ray, the error vanishes to the third order (from the minimum property of  $V$ ) and the next order that enters is the sixth (provided that the inclination of the ray is correct to the first order, a point that requires attention in dealing with thin lenses), so that the value of  $V$  thus calculated is correct to the fourth order. To find  $\alpha, \beta, \gamma, \delta, \epsilon$  we most conveniently use a ray in the principal plane and express it in terms of the initial and final inclinations. On calculating the terms of the fourth order in  $V$  and equating coefficients, we have  $\alpha, \beta, \gamma, \delta, \epsilon$ . This method appears to have no advantages except, perhaps, in the case of the mirror or simple lens.

*The Boiling Point of Sulphur corrected by Reference to New  
Observations on the Absolute Expansion of Mercury.*

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The very accurate observations\* of Eumorfopoulos on the boiling point of sulphur with an air thermometer of the type described by Callendar† gave a final value  $443^{\circ}\cdot58$  C., and appeared to cast some doubt on the value previously assumed, namely  $444^{\circ}\cdot53$  C. on the scale of the constant-pressure air thermometer. It was pointed out, however, by Eumorfopoulos, and also in a note appended by Callendar,‡ that the result depended on the assumption of Regnault's results as recalculated by Broch for the absolute expansion of mercury, and that the final value could be readily corrected when the expansion of mercury had been redetermined by observations then in progress at the Royal College of Science. These observations have now been extended successfully to a temperature of  $300^{\circ}$  C. They will be published in full as soon as the final reductions have been made. But as the results exactly confirm the value previously assumed for the boiling point of sulphur, it appeared desirable to remove this uncertainty at the earliest possible date.

So far as we are aware, no serious attempt has been made to redetermine

\* 'Roy. Soc. Proc.,' A, vol. 81, p. 339, 1908.

† 'Roy. Soc. Proc.,' vol. 50, p. 247, 1891.

‡ 'Roy. Soc. Proc.,' A, vol. 81, p. 363, 1908.

the absolute expansion of mercury above  $100^{\circ}$  C. since the time of Regnault, and it appeared that with modern facilities for accurate measurement a considerable improvement on his work might be effected. The apparatus employed was designed and in part constructed at University College in 1900 while the experiments of Mr. Eumorfopoulos were in progress. The essential points of the design were as follows:

In place of the single pair of vertical tubes 1.5 metres long employed by Regnault, six pairs of tubes, each 2 metres long, were connected in series giving a total length eight times as great.

The difference of level was directly referred to a standard invar scale divided in millimetres with divisions 4 microns wide, and could be read with certainty to 0.001 cm. by means of suitable telescopes with eye-piece micro-meters, giving an order of accuracy of 1 in 20,000 in the fundamental interval. In the course of the work the whole apparatus was twice taken down and re-erected. The extreme differences between independent observations at the same temperature rarely amounted to 1 in 10,000, and it may be assumed that the desired order of accuracy has been substantially attained.

The mean temperatures of the hot and cold columns were readily measured to  $0^{\circ}01$  C. by means of platinum resistance thermometers equal in length to the columns. Regnault employed an air thermometer which could not be relied on beyond a tenth of a degree C., and which also showed a serious change of zero at the conclusion of one series of observations.

The mercury columns were immersed in oil maintained in rapid circulation in well-lagged iron tubes, and were electrically heated by current from a large storage battery, which permitted the attainment of a uniform and constant high temperature for considerable periods, without materially affecting the temperature of the room or disturbing the accuracy of the measurements. Although some of the final reductions have not yet been made it is unlikely that the results at present found will be changed by so much as 1 part in 10,000 of the expansion.

Eumorfopoulos determined the expansion of the bulb of his air thermometer by employing it as a mercury weight thermometer at  $0^{\circ}$ ,  $100^{\circ}$ , and  $184^{\circ}$  C. If  $V$  is the volume of the bulb and  $D$  the density of mercury, his observations for the first Jena glass bulb in March, 1900, were as follows:—

$$V_0D_0 = 1274.824 \text{ grammes weight of mercury filling the bulb at } 0^{\circ} \text{ C.}$$

$$V_0D_0 - V_{100}D_{100} = 19.818 \text{ grammes weight of overflow on heating bulb to } 100^{\circ} \text{ C.}$$

$$V_0D_0 - V_{184}D_{184} = 36.029 \text{ grammes weight of overflow on heating bulb to } 184^{\circ} \text{ C.}$$

In order to obtain a parabolic formula of the type

$$V_t = V_0 \{1 + at + bt(t-100)\}$$

to represent the expansion of the glass, we require only the ratios  $D_0/D_{184}$  and  $D_0/D_{100}$  of the densities of mercury at  $0^\circ$ ,  $100^\circ$ , and  $184^\circ$ . The results of our observations give

$$D_0/D_{100} = 1.018214,$$

$$D_0/D_{184} = 1.033806.$$

From which we obtain the values of the coefficients

$$a = 2385 \times 10^{-8},$$

$$b = 1.31 \times 10^{-8}.$$

The values taken by Eumorfopoulos, assuming Broch's reduction of Regnault's observations, were

$$a = 2387 \times 10^{-8},$$

$$b = 0.42 \times 10^{-8}.$$

The close agreement of the values of  $a$  is to some extent fortuitous, since Regnault's own formula would make  $a$  3 per cent. smaller, whereas Wullner's reduction of Regnault's observations, or Chappuis' independent determinations between  $0^\circ$  and  $100^\circ$ , would make  $a$  2 per cent. larger. These variations, however, are of less importance for the present purpose than the corresponding differences in the coefficient  $b$ .

The correction  $dt$  to be added to the temperature centigrade  $t$  by gas thermometer to allow for the expansion of the bulb is given with sufficient approximation in terms of  $a$  and  $b$  by the formula\*—

$$dt = (a + bT)t(t-100),$$

where  $T = t + 273$ . The correction to be added to the results of Eumorfopoulos for the S.B.P. to allow for the error in the assumed expansion of mercury will therefore be—

$$dt = +0.97^\circ \text{ C.}$$

This would raise his final value of the boiling point of sulphur to

$$444.55^\circ \text{ C.},$$

which is in practically perfect agreement with the value previously assumed.

\* Callendar, 'Phil. Mag.', December, 1899, p. 544.